



# Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

<http://www.tenet.edu/tctm/>

Volume 1 Issue 2

Fall 2003

**Developing Understanding of Fraction  
Concepts: Lessons from Two Teachers**

**A Whole Lotto Education!**

**TCTM Nominations**

Due January 1, 2004

**Building on Children's Thinking to  
Develop Proportional Reasoning**

**Awards Recipients**  
Scholarships, leadership, and more!

**Mathematics, the TEKS, and Literature  
Is it a good fit?**

**Lone Star News**  
Check for upcoming local conferences

**So, Who Invented the Order of  
Operations?**

**Check the Back Cover**  
for your membership card  
and renewal date



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### Editor:

Cynthia L. Schneider  
 234 Preston Hollow  
 New Braunfels, TX 78132  
 cschneider@mail.utexas.edu

### Director of Publication:

Mary Alice Hatchett  
 20172 W Lake PKWY  
 Georgetown, TX 78628-9512  
 mahat@earthlink.net

### Layout and Graphic Designer

Geoffrey Potter  
 <g\_potter@mail.utexas.edu>

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### Call For Articles

The *Texas Mathematics Teacher* seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to the editor with a copy to the director. No author identification should appear on or in the manuscript. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audience should be included. After refereeing, authors will be notified of a publication decision.

Teachers are encouraged to submit articles for *Voices From the Classroom*, including inspirational stories, exemplary lessons, or management tools. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites.

Items for *Lone Star News* include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates

Businesses interested in placing an **advertisement** for mathematics materials should contact Mary Alice Hatchett.

Deadline for submissions: Fall, July 1 Spring, January 1

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# A Whole Lotto Education!

• *Larry Lesser*

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The twice-weekly Texas Lotto Lottery drawings began on November 14, 1992, when I was in the mathematics education Ph.D. program at the University of Texas at Austin. After seeing how many people seemed to have misconceptions about the probabilities and process involved, I decided it was my civic duty as a mathematics educator to offer people a more informed basis about how or whether to play. In 1993, I wrote letters-to-the-editor in local newspapers and created and taught a non-credit, adult continuing education course for the University of Texas Informal Classes called "Lotto Luck!"

The "Lotto Luck!" course was fun to prepare, and it felt good that the course made a difference for the two dozen folks who took it. The course attracted extensive media coverage -- starting from a story spanning 37 column inches in the Austin newspaper (Elliot 1993) all the way to the lead "Dollars and Sense" segment throughout that weekend's Cable News Network (CNN) Headline News! Since then, I've been interviewed by media (e.g., Houston's KTRH-AM, Atlanta's WGST-AM, and Austin's KVUE-TV) on this topic when state lotteries begin new games or amass particularly huge jackpots.

During these unexpected "15 minutes of fame," I've realized anew that good classroom teaching and giving good interviews both require offering examples that allow the listeners to relate the idea to something concrete in their lives or surroundings. Many people of all ages have a hard time visualizing the magnitude of very small or very large numbers. So, let's exercise our number sense as we try to grasp, for example, the probability ( $1/47,784,352$ ) of winning a jackpot under the Texas Lotto configuration initiated on May 7, 2003

(and discussed later in this article):

How about correctly guessing a particular minute from the life of someone almost 91 years old? How about correctly guessing a particular second a student has spent in academic classes by the time she gets to college? How about picking a randomly selected second of music from a radio station's collection of almost 11,000 compact disks? How about correctly guessing a particular inch from the length of the Pecos River? How about dividing the entire state of Texas (264,508 square miles) into pieces of land 3.55 acres each (many individuals or schools or businesses own land this size), and guessing which piece will be selected? How about picking three people at random and seeing if they all were born on April Fools Day? Students and teachers are invited to add to this list by examining their own surroundings -- perhaps they will find that the probability will be similar to that of choosing a particular cubic inch from the volume of the house they live in. Or perhaps they will relate it to choosing a particular letter of a word from a couple of shelves' worth of books!

Because the course was a single two-hour meeting with no mathematical prerequisites, I used familiar low-tech concrete manipulatives such as spinners and dice to illustrate the probability concepts. The spinners were especially simple: we flicked bobby pins or partially unbent paper clips around the points of our upright pencils!

In the full-semester high school and college mathematics courses we teach, however, we can also utilize powerful technology in our explorations as we build up the underlying mathematics step by step. I have discussed (Lesser 1997) how classes can explore

lottery probabilities and expected values using spreadsheet technology, but in this article I offer some probability examples with the common TI-83Plus calculator my high school uses. (Go ahead and get yours now -- I'll wait a moment.)

## Explorations with Drawings

Let's start with how many possible drawings of balls there are. When Lotto Texas started in 1992, 6 balls were drawn without replacement (and without order mattering) from balls numbered 1 to 50. The combination coefficient "50 choose 6" can be evaluated by the TI-83 by entering the following sequence: 50; MATH --> PRB --> nCr; 6; ENTER. We obtain 15,890,700 and the probability a ticket would match the jackpot set of numbers was therefore 1/15,890,700. We can simulate a single drawing (i.e., our own "Quick Pick") with the TI-83 by either the sequence: MATH--> PRB--> randInt(1,50,6); ENTER or the sequence APPS --> ProbSim --> Random Numbers; Set (Numbers: 6; Range: 1-50; Repeat: No); Draw. (I have sometimes used this command when I want an absolutely impartial way of selecting students for a particular task or question.) Students can do many 6-ball simulated drawings and summarize the results. Often they will note and focus on whatever number appears the most often (which motivates some explorations using distributions, as described in the next section).

On July 19, 2000, Lotto Texas was made more challenging by choosing 6 balls from a set numbered 1 to 54. Though the number of balls increased only 8%, the number of combinations increased about 62.5%, which shows how fast combinatorial growth can be. In the bonus ball version of Lotto Texas that started in May 2003, 5 balls are chosen without replacement from white balls numbered 1 to 44, and then a ball is chosen from a set of blue balls numbered 1 to 44. Students can verify that the number of combinations is equivalent to

44 times the quantity "44 choose 5", and that this means the jackpot became 3 times harder to win than the 1992 game.

It was recently reported (Hughes and Marshall, 2003) that by around early November 2003, Texans would be able to play the Mega Millions multistate lottery in which players select five numbers from 1 to 52 and then a sixth number from 1 to 52, resulting in a jackpot probability of 1 in 135,145,920, which is 8.5 times harder to win than the 1992 Lotto Texas game! By the way, the rationale of having the jackpot odds that increase in difficulty is that jackpots will be won less often, therefore rolling over into larger jackpots more often, presumably tempting more people to play (and generating more money for Texas education!).

## Explorations with Distributions

Recall that for discrete probability distributions, the pdf (probability density function) gives you the probability that a random variable equals a specific value, while the cdf (cumulative distribution function) gives the probability a random variable is less than or equal to a specific value. While these concepts can be very abstract to students in their first statistics course, lottery drawings offer a concrete real-world application for many of the most commonly encountered discrete random variables. (Actually, drawing one number from a set of numbered balls has already illustrated the discrete uniform distribution!) We will keep this section concise, but the distributions that follow are all also in the TI-83Plus calculator manual (with syntax examples, defining formulas, and screen shots) and in standard statistics textbooks:

A binomial random variable is the number (x) of successes when there is a fixed number (n) of independent trials, each of which has a success probability (p). Entering values for n, p, and x (in that order) into the TI-83's binomcdf or binompdf commands (via the 2nd DISTR keys) can be used to investigate

how rare it is for a particular lottery number to occur so frequently out of a fixed number of drawings. For example, in the 1992 50-ball game, the probability of the number 17 occurring exactly 3 times in 20 6-ball drawings would be  $\text{binompdf}(20, .12, 3)$ , or about 22.4%.

A geometric random variable is the number  $x$  of trials (each of which is independent and has probability  $p$  of success) for the first success to occur. The  $\text{geompdf}$  and  $\text{geomcdf}$  commands (inserting numbers for the values of  $p$  and  $x$ ) can therefore be used to explore the probability that it takes a particular number of drawings before a drawing includes a particular number (or, for that matter, to finally win a jackpot). For the 1992 game, the probability of no more than 50 drawings needed for the first occurrence of 17 is:  $\text{geomcdf}(.12, 50)$ , which is about 99.8%.

A Poisson random variable is the number  $x$  of occurrences of an event during an interval with a known average (mean) number of occurrences per interval of that length. If players choose number combinations randomly (e.g., Quick Pick), then we can use the Poisson distribution to find the probability of, for example, exactly one person winning the jackpot. If the jackpot probability is  $p$  and  $t$  tickets are sold, then the number of jackpot winners is a binomial random variable with expected value  $pt$ . Using the value of  $p$  for a particular lottery game (as discussed in the Exploration with Drawings section), try out various values of  $t$  to see when the probability of winning a jackpot all by yourself is maximized. On the TI-83 use this sequence: 2nd DISTR;  $\text{poissonpdf}(\text{your value for } p \text{ times } t; 1)$ .

And to explore the randomness of all the past Lotto numbers chosen (they may be found via the Texas Lottery website), we can do descriptive statistics and a chi-square test, as did Lamb et.al. (1994). For extra credit, the hypergeometric distribution is not in the TI-83Plus calculator, but can be assembled as an arithmetic combination of three uses of the MATH  $\rightarrow$  PRB  $\rightarrow$  nCr

sequence. The hypergeometric distribution allows students to verify the playslip's printed odds of winning each of the cash prizes, not just the grand jackpot. Students may also be interested to verify the surprising situation of a national lottery in which the fifth-highest cash prize actually turned out to be more probable than the sixth prize (Helman 2003)!

## A Musical Conclusion

Let's close with a very different mode of outreach education I've employed on this topic -- a math song! I first published this parody lyric in the Winter 2002 issue of *STATS*, and performed it most recently at Lesser (2003). The lyric may be sung to the tune of the Don Schlitz song of the same title (which yielded Kenny Rogers a #1 country hit, signature song, and television mini-series). It helps students distinguish the small bit of strategy (e.g., Henze and Riedwyl 1998) from the multitude of misconceptions:

### "The Gambler"

(lyrics (c) 2001 Lawrence M. Lesser;  
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On a warm summer's evenin', on a train bound for nowhere,  
I met up with a gambler -- we were both too tired to sleep.  
So he told me how he planned winnin' lottery prizes  
'Til, as a math teacher, I just had to speak:

"Son, I've made a life out of readin' students' faces,  
Checkin' comprehension by the way they held their eyes.  
And I can see your blackboard is erased in some places--  
Give me some peanuts and I'll give ya some advice.

First, your instant scratch-off tickets give 1 in 5 chances,  
But that don't mean that 1 in 5 will win.  
'Cause ev'ry ticket's sep'rate, like a new flip of a coin:  
It has no mem'ry how your wallet's gotten thin!

And you track those weekly draws, you say ya got a system--  
You call some numbers "hot", you deem others "due";  
But I insist, they each have the same chance--  
If you're gonna play the game, boy, ya gotta know what's true!  
(Chorus)

You gotta know when you pick 'em,  
What's superstition,  
And where strategy is there to be had,  
Or you'll learn why  
Lotteries seem like  
Tax on folks who don't know much math!



Now all sets of numbers are equally unlikely,  
 More rare than death by lightning, still there's somethin' you  
 should know;  
 If you should happen to win that big jackpot,  
 You'll win more money if you picked it all alone!

So avoid those numbers that more folks are playin':  
 Like sevens and birthdays and sequences, too.  
 'Til this song gets famous, you'll have the advantage--  
 Maybe you'll thank me with a share of your loot!"  
 (Repeat Chorus)



Larry Lesser, Ph.D. • <llesser@EmeryWeiner.org>  
 Head of Mathematics Department • Emery High School,  
 Houston, TX

References

Elliot, D. (1993, August 28), Professor seeks to even  
 odds of lottery. *Austin American-Statesman*, B1,  
 B3.

Helman, D. (Summer 2003). A lottery misfortune.  
*Teaching Statistics*, 25(2), 40-41.  
 Henze, N. and Riedwyl, H. (1998). *How to win more:  
 Strategies for increasing a lottery win*. Natick, MA:  
 A.K. Peters.  
 Hughes, P.R. and T. Marshall (August 6, 2003).  
 Texas bets on Mega Millions. *Houston Chronicle*,  
 102 (297), p. 1A, 10A.  
 Lamb, J.; Huffstutler, R.; Brock, A.; Aslan, F.  
 (January 1994). A statistical analysis of the Texas  
 Lottery. *Texas Mathematics Teacher*, 41(1), 4-17.  
 Lesser, L. (1997). Exploring lotteries with Excel.  
*Spreadsheet User*, 4(2), 4-7.  
 Lesser, L. (2003). *Formula for a Hit: Using Songs  
 in Mathematics Class*. Presentation at 50th  
 annual Conference for the Advancement of  
 Mathematics Teaching, Houston, TX.

Texas Lottery web site: <http://www.txlottery.org/>

## CAMTership Application

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**Eligibility:** Six \$200 CAMTerships will be awarded to teachers with five or fewer years teachig experience who are members of TCTM and have not attended CAMT before. CAMTerships will be awarded to teachers in each of the following grade levels: K - 4, 5 - 8, and 9 - 12. Winners will be determined by random drawing of names and will be notified by June 1, 2004. Winners will be asked to work for two hours at registration or the NCTM material sales booth and will be TCTM's guest at our breakfast, where the checks will be presented. Good luck!

**Deadline:** May 1, 2004

Name: \_\_\_\_\_  
Last First Middle

Address: \_\_\_\_\_  
Number and street Apt. number

\_\_\_\_\_ City Zip Code

(\_\_\_\_) \_\_\_\_\_ (\_\_\_\_) \_\_\_\_\_ < \_\_\_\_\_ >  
Home Phone Work Phone Email Address

Are you a member of TCTM?  yes  no      Have you attended CAMT before?  yes  no  
**Note:** If you are not a member of TCTM, you must enclose \$13 with this application to apply for membership.  
 How long have you been teaching? \_\_\_\_\_ Describe your primary teaching responsibilities

Send your completed application to:  
 by mail: **Cynthia Schneider,** by fax: (512) 232-1855 by email:  
**234 Preston Hollow,** ATTN: Cynthia Schneider <[cschneider@mail.utexas.edu](mailto:cschneider@mail.utexas.edu)>  
**New Braunfels, TX 78132**