for my first in-person BRIDGES paper, I went big!

“On and On: Infinity in Song”

“On and On: Infinity in Song”

Professor Lesser

“On and On: Infinity in Song”

Larry Lesser

The University of Texas at El Paso

My backgrounds

- Became math major & songwriter same year (of college)
- State/national awards as a STEM educator
- 20 awards in national STEM song contests

(and awards for non-STEM songs in statewide contests)

- Math-and-music outreach: LarryLesser.com/mathemusician
- Interest in infinity -- my archived talk on infinity in math and religion

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aspects of songs that could involve infinity

- Lyrics
- Music structure
- Recording process

popular-style songs with lyrics

- **MATHEMATICAL SONGS:**

written (by math instructors/educators)
with main goal of math learning/appreciation

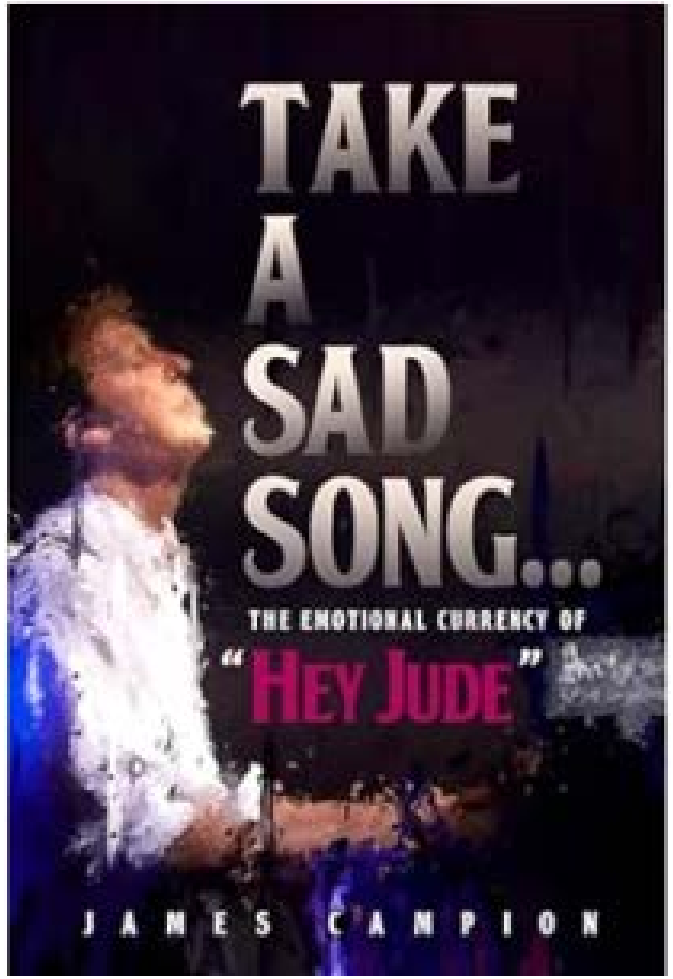
- **NON-MATHEMATICAL SONGS:**

commercially released in the music business
without the above main goal

Seeking infinity in musical structure:
"Infinity Section" (as opposed to "Expectancy Section")

- repeated musical phrase that opens up the song with new material

rarely does a song have an entire book on it…



Take a Sad Song: The Emotional Currency of "Hey Jude"

Hardcover – June 1, 2022

by James Campion (Author)

4.8 ★★★★★ 39 ratings

See all formats and editions

In *Take a Sad Song: The Emotional Currency of "Hey Jude,"* James Campion dives deeply into the song's origins, recording, visual presentation, impact, and eventual influence, while also discovering what makes "Hey Jude" a classic musical expression of personal comfort and societal unity conceived by a master songwriter, Paul McCartney. Within its melodic brilliance and lyrical touchstones of empathy and nostalgia resides McCartney's personal and professional relationship with his childhood friend and songwriting partner, John Lennon, and their simultaneous pursuit of the women who would complete them. There are also clues to the growing turmoil within the Beatles and their splintering generation scarred by war, assassination, and virulent protest.

Campion's journey into the song includes the insights of experts in the fields of musicology, sociology, philosophy, psychology, and history. Campion also reveals commentary from noted Beatles authors, biographers, music historians, and journalists and, finally, a peek into the craft of songwriting from a host of talented composers across several generations.

Take a Sad Song is a tribute to how a song can define, inspire, and affect us in ways we do not always fully comprehend, as well as a celebration of a truly amazing track in the Beatles canon that reveals one band's genius and underscores its lasting voice in our cultural and musical landscape.

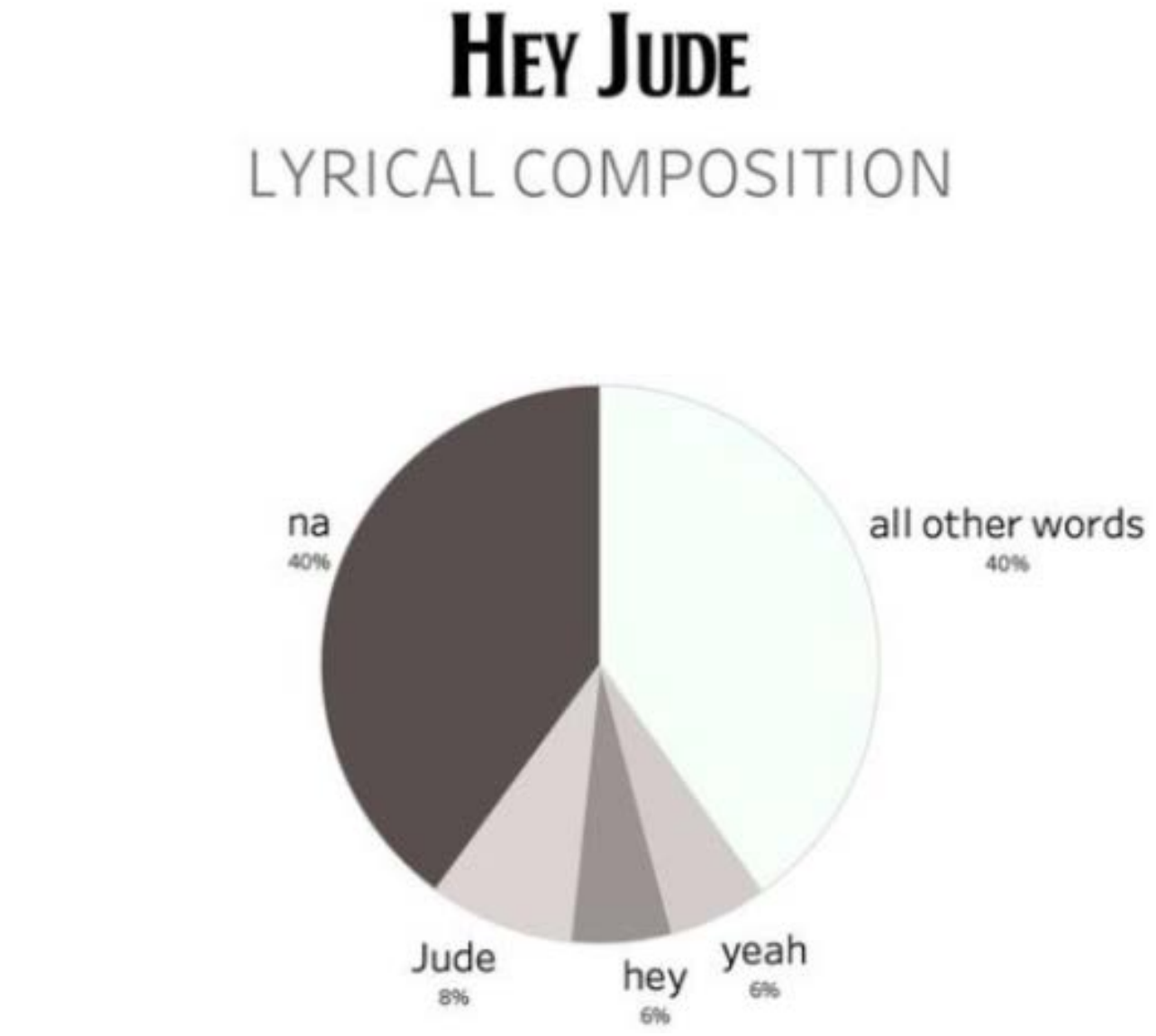
Seeking infinity in musical structure:
"Infinity Section" (as opposed to "Expectancy Section")

- repeated musical phrase that opens up the song with new material
- Classic example: 4-bar loop at end of The Beatles' "Hey Jude"

Simon shows you maps · Follow
a day ago ·

Na, Jude, hey, yeah. These four words make up 60 per cent of all "Hey Jude" lyrics.

Thanks to @boknowsdata we now won't ever forget this important information 😉



made by Bo McCready
@boknowsdata

…or the whole song is what repeats!

Shari Lewis children's marching song "The Song That Doesn't End"

(https://en.wikipedia.org/wiki/The_Song_That_Doesn%27t_End)

This is the song that doesn't end

Yes, it goes on and on, my friend

Some people started singing it not knowing what it was

And they'll continue singing it forever just because…

Seeking infinity in the recording process

https://pixabay.com/photos/old-phone-rotary-rotary-phone-956455/

- end of 1st verse (at 0:55) from “One Stage Before” by Al Stewart:

“But where or when we met like this last time I just don’t know:

It’s like a chord that rings and never dies….. *for infinity…..*

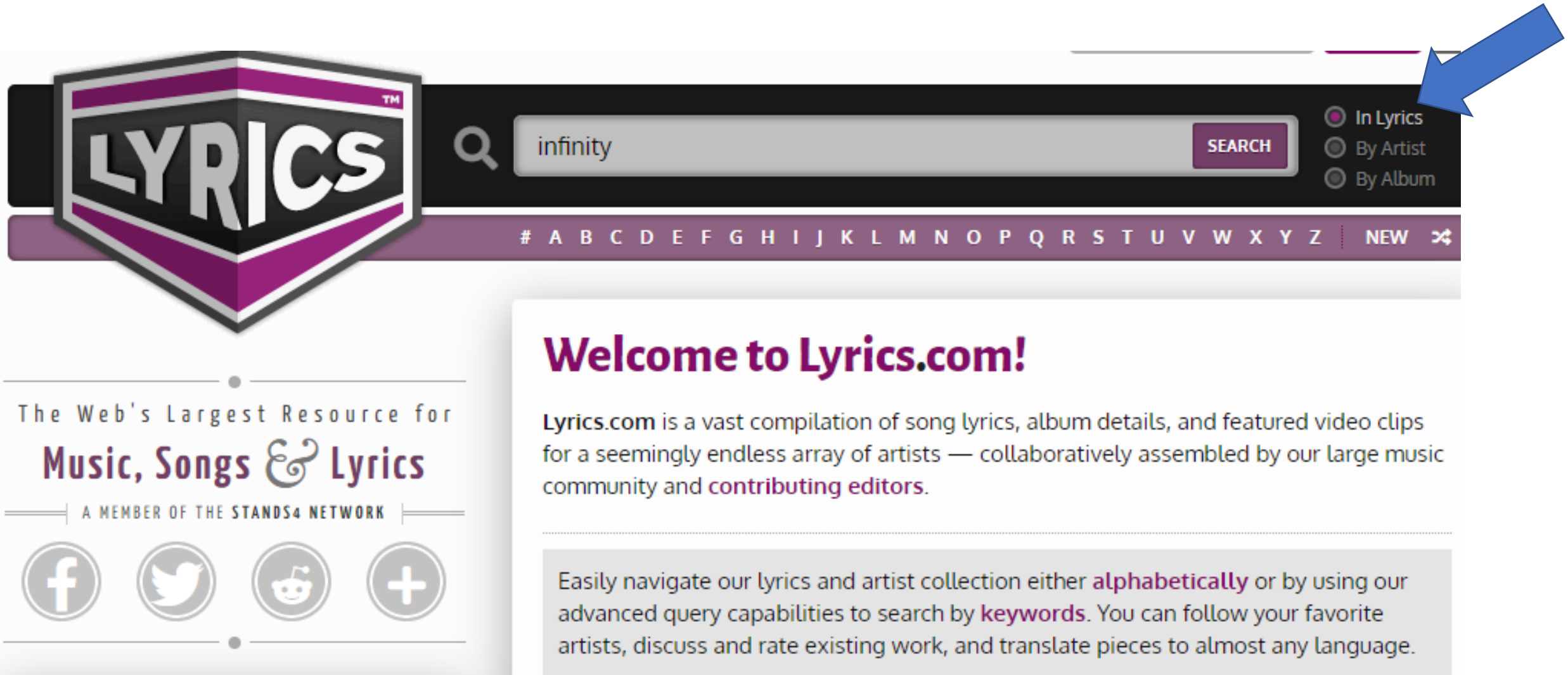
https://www.youtube.com/watch?v=gt7yEnuXxeY&t=55s

- Studio “telephone” effect (bandpass filter leaves only a narrow middle spectrum around 1-2 kHz) to make vocals sound distant

Seeking infinity in lyrics

- the actual title (e.g., by Mariah Carey or One Direction)
- a word *in* the title
- a word in the lyric, but not the title (e.g., “One Stage Before”)
- an *idea* in the lyric, without using the word *infinity*

3/31/2024 Lyrics.com search found 12,743 lyrics with “infinity”



themes I identified from a sample
(in descending frequency)

Theme	Sample lyric phrase
Romantic love and/or forever	“I love you to[till] infinity” ; “you’re my infinity”
State of mind or personification	“I am infinity”
Idealized sense of home/place	“take me to infinity”; “I surrender to the fields of infinity”
Sense of transcendence	“infinity and beyond” ; “reach beyond infinity”
Divine attribute	“Lord, You are infinity”

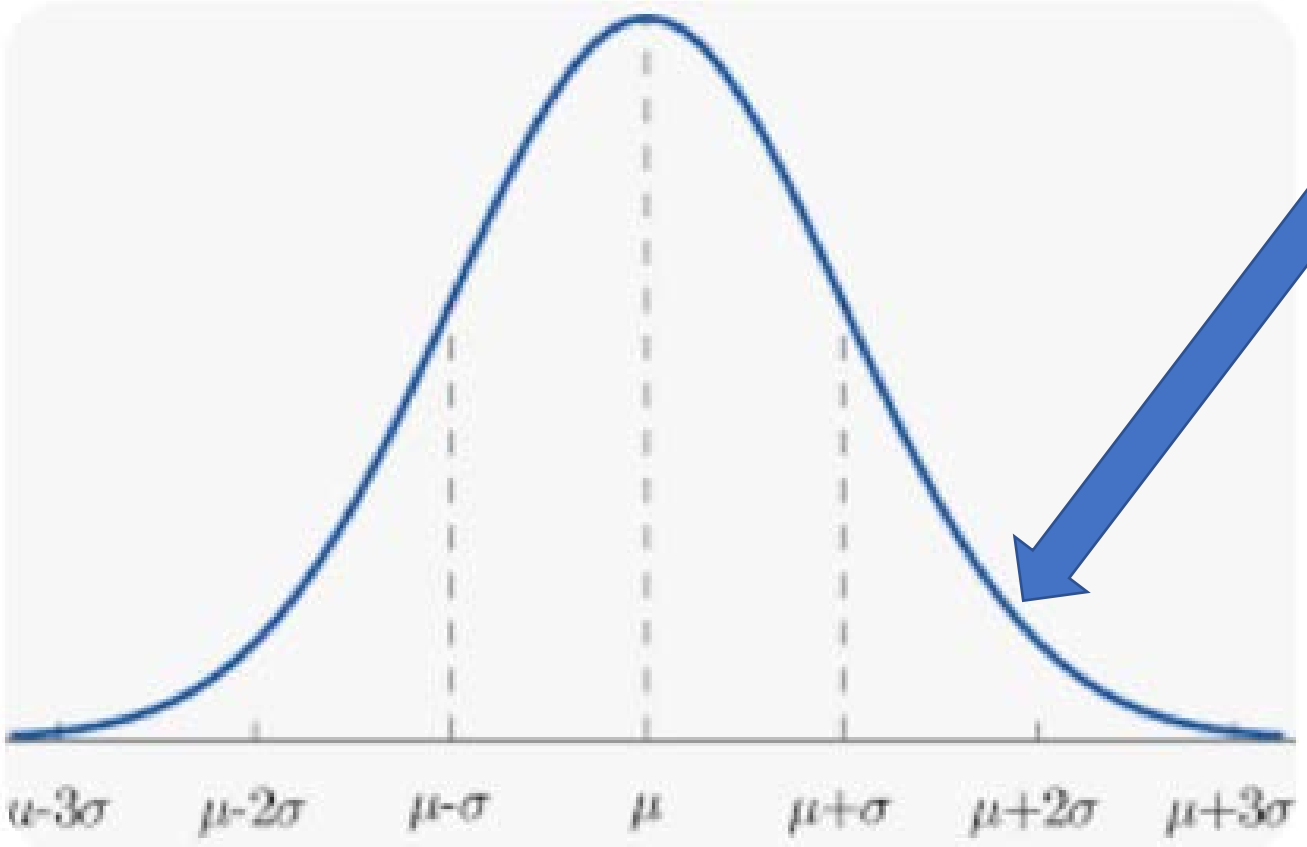
Let’s share examples of non-math songs with math concepts evoking infinity

Math concept/object	Artist (song title)
Asymptotic behavior of graph	David Wilcox (“Down Here”); Stephanie Chou (“Asymptote”); Nicole Campbell (“Horizontal Asymptote”)
Möbius strip	Nelly Furtado (“Hey Man!”)
Fractals	Jonathan Coulton (“Mandelbrot Set”)
Perspective (point at infinity)	Larry Lesser (“Rails”)
Infinite (geometric) series	Clayton Jokythur (“Zeno’s Paradox”)
Cardinality	Christine Kane (“Tucson”)

“tails” of a graph in David Wilcox’s “Down Here”

So you come for a visit, but you lose your nerve

Out at the edges of the bell-shaped curve.



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

asymptote behavior in relationship songs….

Stephanie Chou (“Asymptote”)

I wanted to be near you; you want to be free.
The more we grew, the closer we came to what could be.
When will I be your one and only? Was it just a silly dream?
You wanted me to hear you. This is all you can be.
I know you wish you could be that man for me.
Have you just reached your limit, baby?
Well I guess in time, we'll see.

Nicole Campbell (“Horizontal Asymptote”)

I'm always getting closer to you
Arbitrarily close to you
But no matter how close I go, I'll never get to touch you,
You horizontal asymptote

Möbius strip in Nelly Furtado's "Hey Man!"

photo: David Benbennick

Hey man, don't look so angry
You're real close to figuring me out
We are a part of a circle, it's like a Möbius strip
And it goes 'round and 'round until it loses a link

fractals in Jonathan Coulton's "Mandelbrot Set"
(see Sarah Glaz 2010 Bridges paper)

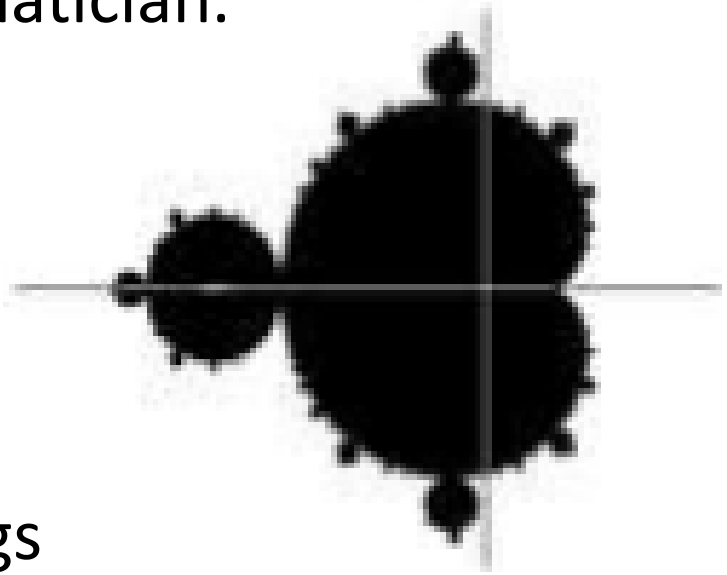
"Pathological monsters!" cried the terrified mathematician.

"Every one of them is a splinter in my eye

I hate the Peano Space and the Koch Curve

I fear the Cantor Ternary Set

And the Sierpinski Gasket makes me want to cry"



And a million miles away a butterfly flapped its wings

On a cold November day, a man named Benoît Mandelbrot was born

vanishing point at infinity in my “Rails”

https://pixabay.com/photos/railway-rails-track-train-2439189/

Like these rails,

looks like we’ll meet down the line.

Love that fails

seems to just need time.

The way you set your eyes on

that horizon

shows me love is blind.

infinite geometric series in Clayton Jokythur's "Zeno's Paradox"

https://pixabay.com/photos/tortoise-reptile-giant-tortoise-2815539/

Imagine a tortoise like me
And Achilles that Greek man
Hero, you might say, yeah you know he fast
And I ask you, in a race with him,
    what chance would I stand
So he wants to be a nice guy
Gives me a head start, then I win, why?

'Cause 'til he catches up with me
I will already be ahead, see?
And so on and so on
And he'll look like a moron
For losing to a tortoise
That's right my boys -- Zeno's paradox!
And this one I am not explaining
Because paradoxical as it is,
I'm the one who's winning

cardinality in Christine Kane's "Tucson"

Way out west a girl could lose her given sense
In all the truth that old desert sky can tell
That horizon's just too much for me
But if I could count infinity, I'd know the desert well

mathematical or non-mathematical song?

“Infinity” from GRAMMY-winning 2008 children’s album
Here Come the 123s by They Might Be Giants

Infinity, infinity, how can I reach infinity?
It's the highest number there could possibly be
But I'm gonna reach infinity
If I chase my tail and follow its tracks
If I run around in circles, I'll never make it back
Oh, figure of 8 goes around, around, around,….

some overview of mathematical songs

- CAUSEweb.org/fun has curated, searchable set of 200 statistics songs
- CAUSEweb.org/voices (**V**irtual **O**ngoing **I**nterdisciplinary **C**ollaborations on **E**ducating with **S**ong) has 100+ archived presentations from 5+ years' worth of conferences on song in STEM
- my 2024 paper in current issue of *PRIMUS* has references on educational effectiveness of mathematical song
- my math/stat song page: LarryLesser.com/mathemusician

Classroom songs usually….

- are short(ened) without long intros and solos
- have repeated choruses
- are parodies (easier to write/learn with tune already familiar)

John Dougherty's "The Recursion Song"

tune: children's song "It's raining, it's pouring"

Keep singing this…..

Recursion, recursion, we'll sing about recursion

Until the case has reached its base, we'll sing about recursion

John Dougherty's "The Recursion Song"

tune: children's song "It's raining, it's pouring"

Recursion, recursion, we'll sing about recursion

~~Until the case has reached its base, we'll sing about recursion~~

….then finally end with this:

Now the case has reached its base, ending the recursion!

unending sequence of (nonrepeating) digits….

CHORUS of my “American Pi”

(tune: Don McLean #1 hit “American Pie”)

Find, find, the value of pi – starts 3 point 1 4 1 5 9.

A good ol’ fraction you may hope to define,

But the decimal never dies, never repeats or dies….

Infinite set of (prime) numbers

Verse #1 from my “Prime After Prime”

(tune: Cyndi Lauper #1 hit “Time After Time”)

Euclid, he formed the product of “every prime”

And 1 plus that product has a factor that’s another prime!

So we see the big bottom line – there is no final prime!

If you want, you can look and you’ll be finding: prime after prime.

If you want, you can look and they’ll be waiting: prime after prime.

Fractal verse in my “From a Distance”

(tune: Julie Gold’s same-titled GRAMMY-winning hit for Bette Midler)

(Wikipedia image by Chas zzz brown)

From a distance, some curves look the same

as they do when we are close

From a distance, patterns repeat

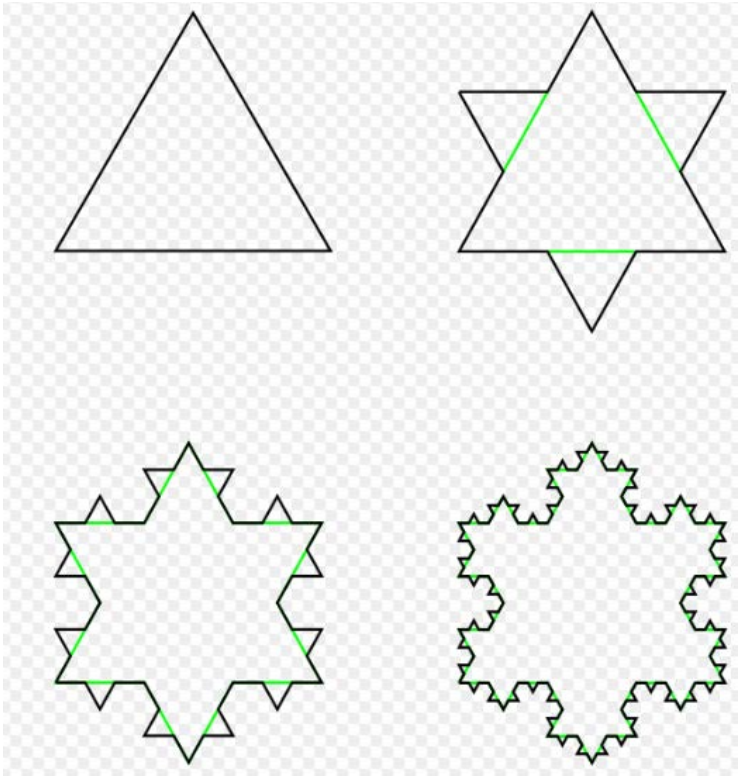
along a cloud or coast.

From a distance, there is similarity

and it echoes at every scale.

Let’s find what makes those Koch snowflakes

and follow the fractal trail.



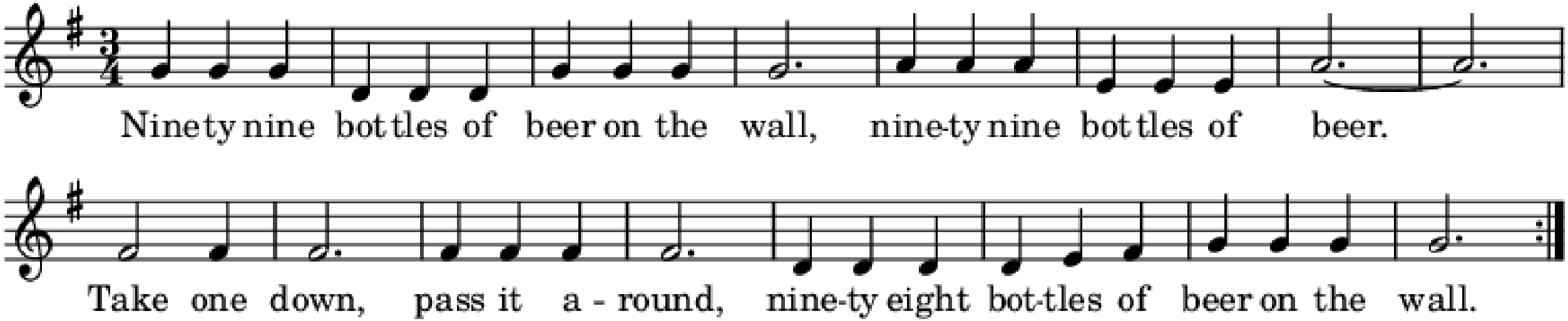
Infinite hotel metaphor from David Hilbert lecture a century ago

from my “Hotel Called Infinity” (tune: the Eagles’ #1 hit “Hotel California”):

CHORUS: Welcome to the hotel called Infinity
Where every room is full (every room is full) yet there’s room for more
Yeah, plenty of room at the hotel called Infinity—
Move ‘em down the floor (move ‘em down the floor) to make room for more

verse: My mind got more twisted when I saw a bus without end
With an infinite number of riders coming up to check in.
“Relax,” said the nightman. “You each will move
To the double of your room number: that frees the odd-numbered rooms.”

“99 bottles of beer on the wall”

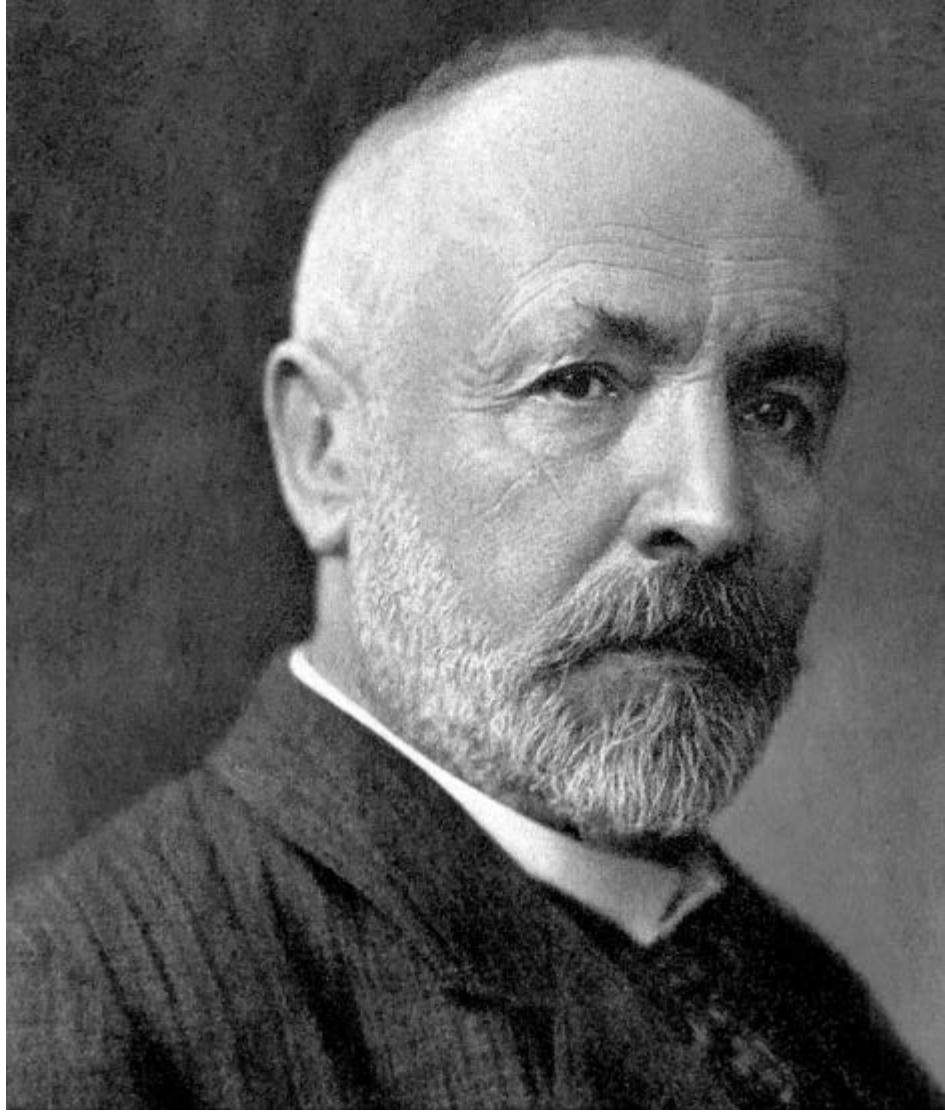


99 bottles of beer on the wall,
99 bottles of beer.
Take 1 down, pass it around:
98 bottles of beer on the wall!

“~~99~~ $\aleph_o$ bottles of beer on the wall” (Renteln & Dundes, 2005)

$\aleph_o$ bottles of beer on the wall,
$\aleph_o$ bottles of beer.
Take 1 down, pass it around:
$\aleph_o$ bottles of beer on the wall!

from my “Cantor’s Coat”



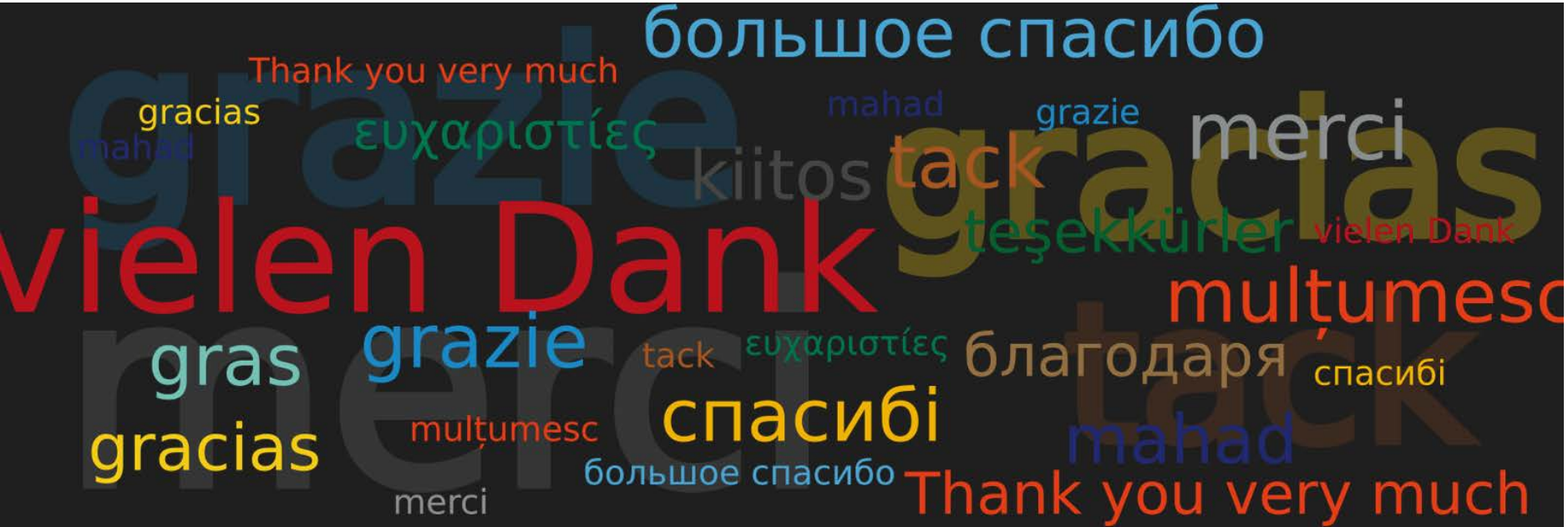
He picked his path
the freedom of math
but met with wrath
For counting infinities
real as the Trinity,
uncountably vast.
Cantor believed this was Divine Plan
But his mentor said, “God made whole numbers, the rest is by man!”

I look forward to discussion!

“On and On: Infinity in Song”

Larry Lesser, The University of Texas at El Paso

LarryLesser.com Lesser@utep.edu



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